

This is a draft and will be cleaned up before assigning to students. The following would be assigned as a computer lab and unit project to a high school geometry class before a unit on parallel lines and polygon angles, and would be due at the end of the unit. The day in the computer lab students would complete all steps before Questions and Conjectures, and just the measurements in Questions and Conjectures. A day or so after the lab the final sketch would be posted on my web page as a JavaSketchpad object that students could use in completing the project. (It, and this document, are actually posted now; see <http://home.comcast.net/~sks23cu#TriangleAngles>)

----- Handout -----

Review: From Help in Geometer's Sketchpad:

The Toolbox appears on the left of the screen when you start Sketchpad, and includes six tools.



- **Selection Arrow** tools: Use this tool to select and drag objects in your sketch. The three variations of the tool allow you to drag-translate (move), drag-rotate (turn), and drag-dilate (shrink or grow) objects.
- **Point** tool: Use this tool to construct points.
- **Compass** tool: Use this tool to construct circles.
- **Straightedge** tools: Use this tool to construct straight objects. The three variations of the tool allow you to construct segments, rays, and lines.
- **Text** tool: Use this tool to create and edit text and labels.
- **Custom** tools icon: Use this icon to define, use, and manage custom tools.

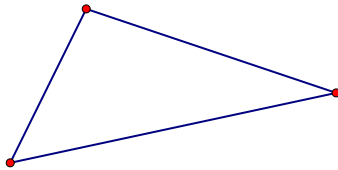
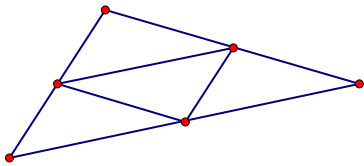
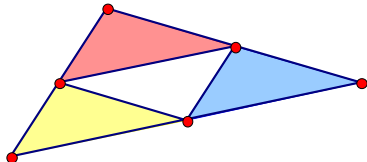
Lab Overview

In this lab you will investigate the relationships in a triangle between various interior and exterior angles and sums of measures of those angles. In particular, you will find:

- The sum of the measures of the three interior angles of a triangle;
- The measure relationships of an exterior angle of a triangle and its two remote interior angles;
- The sum of the measures of the exterior angles of a triangle; and
- The relationships of angles around intersections of parallel lines and transversals.

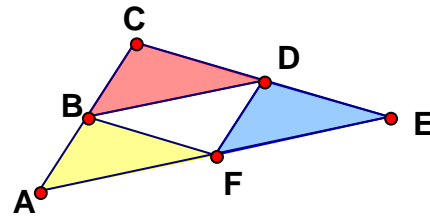
Basic Setup

We need a triangle whose angle relationships are easy to identify by sight. To do so we will construct a triangle with differently colored angles.

- Use the Point Tool to draw three (3) points anywhere in the Sketchpad window.
- Use the Selection Tool to select all three points by clicking on each one to make it bold.
- Pull down the Construct Menu at the top of the window and select Segments. You should now have a triangle like the one at the right.
- Use the Selection Tool to select all three segments by clicking on each one to make it bold. Be careful to NOT select the points.
- Pull down the Construct Menu at the top of the window and select Midpoints, and immediately pull down the Construct Menu again and select Segments. You should now have a figure like the one at the right.
- Construct Interiors to each of the smaller triangles, except for the center one. Do this by Selecting the vertices of one of them, pulling down the Construct Menu and selecting Triangle Interior. Repeat for each of the other small triangles, except the center one. Change the color of the interiors to match the figure on the right (right click on an interior, select Color, and choose the proper color).

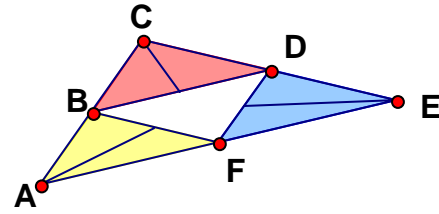
Lab Project: Triangle Angles

7. Starting at the bottom left point and proceeding clockwise around the outside of the large triangle, select all the points. Pull down the Display Menu and select Show Labels. Your figure should look like the one at the right.



If your labels are too small or too light, pull down the Display Menu and select Show Text Palette. It will appear at the bottom of screen. With all the points still selected, use the Text Palette to make their labels bold (**B** button) and a larger size (it's 14 in the figure above).

8. Select only the sides of the small center triangle and Construct Midpoints on them. Then Construct Segments between each of those midpoints and the vertex across its colored triangle. Hide the new midpoints (select only them, pull down the Display Menu, select Hide Points). Your figure should look like that on the right.

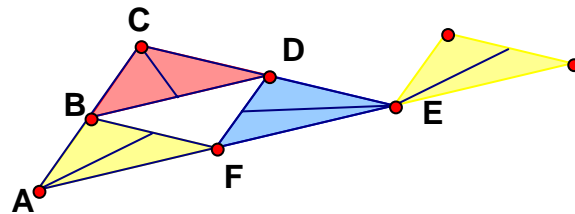


Try dragging individual points, segments, or interiors to see how the parts change.

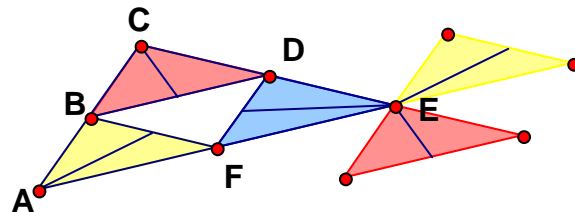
Transform Angles

We will now slide and rotate copies of the smaller triangles to better show angle relationships.

9. In order, select points A and E; then pull down the Transform Menu and select Mark Vector.
10. Select only points B and F, the yellow interior, and the segment inside it. Pull down the Transform Menu and select Translate; click Translate in the dialog box that appears. Your figure should look like the one at the right.

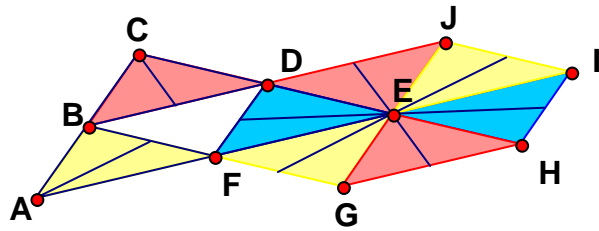


11. Repeat those steps to translate the red triangle along segment CE. Your figure should look like the one at the right.

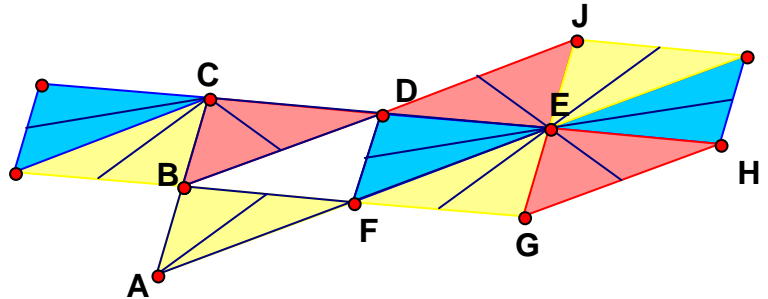


Lab Project: Triangle Angles

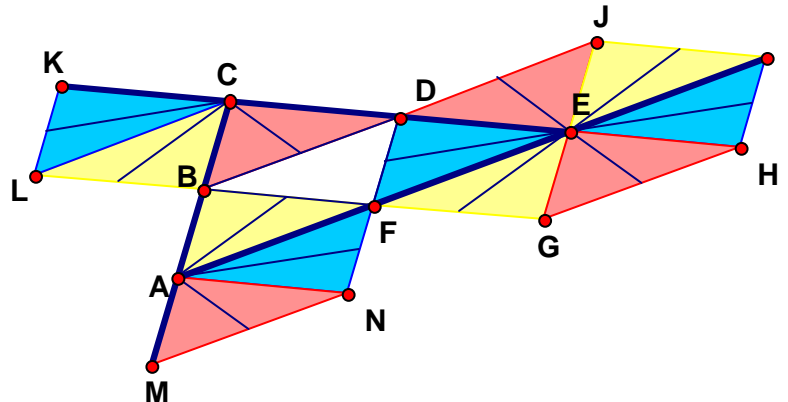
12. Double click point E (to mark it as the center of rotation). Select only the three interiors touching point E and the segments in them. Pull down the Transform Menu and select Rotate; type 180 in the degrees box and click on Rotate. Your figure should look like the one at the right. Use the Text Tool to label points and reposition the labels.



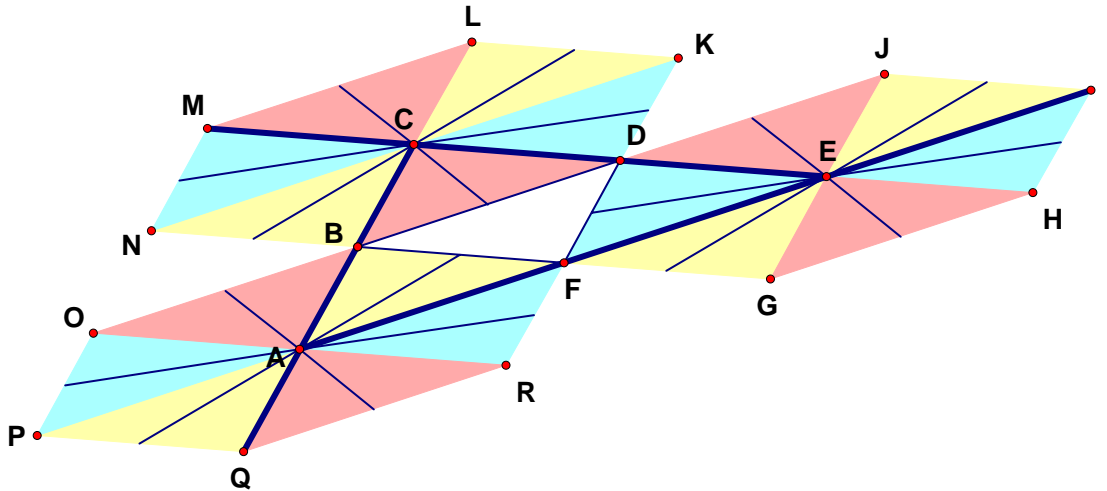
13. Select points E and C, pull down the Transform Menu and select Mark Vector. Select points D and F, the blue and yellow interiors between points D and G, and the segments inside the two interiors. Pull down the Transform Menu, select Translate, and click Translate in the dialog box. Your figure should look like this:



14. Select points E and A, pull down the Transform Menu and select Mark Vector. Select points G and H, the red and blue interiors between points G and I, and the segments inside the two interiors. Pull down the Transform Menu, select Translate, and click Translate in the dialog box. After labeling the points and drawing thick segments for the extended sides, your figure should look like this:



15. To completely define the angles around each vertex of triangle ACE, rotate the colored angles on one side of straight angle $\angle DCK$ to the other side, and do the same to straight angle $\angle BAM$. Your figure should look like this:



Try dragging individual points, segments, or interiors to see how the parts change.

Questions and Conjectures

N.B.: For triangle ACE:

- The *interior* angles are $\angle ACE$, $\angle CEA$, and $\angle EAC$.
 - The *exterior* angles are $\angle IEM$, $\angle MCQ$, and $\angle QAI$.
16. Are points A, F, E, and I collinear? Explain why, based on the construction methods. Measure $\angle AEI$ (select points A, E, and I, then pull down the Measure Menu and select Angle) and drag various points around to see how the measure changes. Do the results confirm your collinearity assertion? Explain.
17. **Sum Of The Interior Angles Of A Triangle.**
- What conjecture about the sum of the measures of the interior angles of triangle ACE could you make by referring to your sketch? Explain why, based on the construction methods.
 - Measure all the interior angles, then Calculate their sum (pull down the Measure Menu, select Calculate and follow on-screen instructions to add the three measures). Drag various points around to see how the measures change. Do the results confirm your conjecture? Explain.

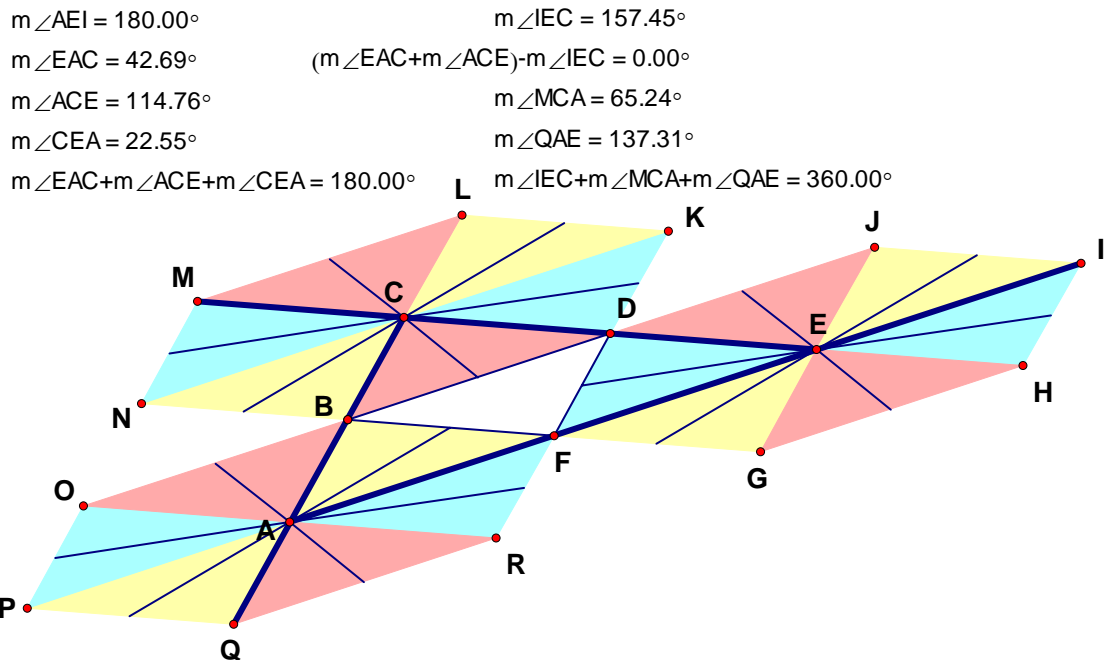
18. Exterior Angle vs. Remote Interior Angles Of A Triangle.

- a. From your sketch, what conjecture could you make about the measure of exterior $\angle IED$ and its *remote interior angles* ($\angle FAB$ and $\angle BCD$)? Explain why, based on the construction methods.
- b. Measure the three angles and Calculate any sums necessary to demonstrate your conjecture. Drag various points around to see how the measures change. Do the results confirm your conjecture? Explain.

19. Sum Of The Exterior Angles Of A Triangle.

- a. What conjecture about the sum of the measures of the exterior angles of triangle ACE could you make by referring to your sketch? Explain why, based on the construction methods.
- b. Measure all the exterior angles and Calculate their sum. Drag various points around to see if the measures change. Do the results confirm your conjecture? Explain.

Your sketch might now look like this:



N.B.: A line that intersects two or more other lines is called a *transversal*; e.g., lines AE and CE are transversals of lines GE and AC. Considering any pair of lines intersected by a transversal, the four angles that are between the lines intersected by the transversal are called *interior angles*, the other four are called *exterior angles*. Two angles, one from each intersection, that are on the same side of the transversal are called *same-side angles*, otherwise they're called *alternate angles*. Same-side angles that are not both interior are called *corresponding angles*.

20. Explain why, based on your sketch and its construction methods, the following is true:

- a. Points G, E, and J are colinear, and line GJ is parallel to line AC;
- b. Corresponding angles are congruent (have equal measures);
- c. Alternate exterior angles are congruent;
- d. Alternate interior angles are congruent;
- e. Same-side exterior angles are supplementary; and
- f. Same-side interior angles are supplementary.